A Measurement of the Cosmological Constant Using Elliptical Galaxies as Strong Gravitational Lenses

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ABSTRACT

We have identified seven (field) elliptical galaxies acting as strong gravitational lenses and have used them to measure cosmological parameters. To find the most likely value for Ω_m ($= \Omega_{matter}$) and Λ, we have used the combined probabilities of these lens systems having the observed critical radii (or image deflection) for the measured or estimated values of lens redshifts, source redshifts, and lens magnitudes. Our measurement gives $\Lambda = 0.64^{+0.15}_{-0.26}$ if $\Omega_m + \Lambda = 1$, and the $\Omega_m = 1$ model is excluded at the 97 % confidence level. We also find, at the 68 % ($\Omega = 0$) – 82 % ($\Omega = 0.3$) confidence level, that an open universe is less likely than a flat universe with non-zero Λ. Except for the possibility of strong perturbations due to cluster potentials and the systematic overestimate of the lens magnitudes, other possible systematic errors do not seem to influence our results strongly: correction of possible systematic errors seems to increase the significance of the result in favor of a non-zero Λ model.

Subject headings: cosmology: theory - cosmology: observations - cosmology: gravitational lensing - galaxies: evolution

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1. Introduction

Recently, non-zero cosmological constant (Λ) models have found increased popularity (e.g., Ostriker & Steinhardt 1995) owing to the problem of the age discrepancy implied by the latest Hubble Space Telescope (HST) measurements of the Hubble constant: the ages of globular clusters are apparently larger than the age of the universe predicted by the standard $\Omega_m = 1$ model which is favored by standard inflationary theory (e.g., Freedman et al. 1994; for a brief overview of these arguments, see Rees 1996).

In order to measure Λ, it has been suggested that strong gravitational lenses might be used, i.e. isolated galaxies or clusters of galaxies for which the gravitational potential results in multiple

imaging of a background object (Paczynski & Gorski 1981; Alock & Anderson 1986; Gott, Park, & Lee 1989). Following these suggestions, the use of the lens number counts (or the optical depth) was advocated since this is very sensitive to Λ (Turner 1990; Fukugita, Futamase, & Kasai 1990; Fukugita et al. 1992). Maoz & Rix (1993) and Kochanek (1995) have applied this method, obtaining upper limits of $\Lambda \lesssim 0.7$.

Kochanek (1992) has also suggested the lens redshift method, which, compared with the method based on lens counts, requires less presumptions about the properties of lenses and sources, properties which might bias the lens counts considerably (Helbig & Kayser 1995; Kochanek 1992; Fukugita & Peebles 1995). Taking into account the selection effects which were neglected in his early study (Kochanek 1992: for a discussion on this selection effect, see Helbig & Kayser 1995), Kochanek (1995) finds $\Lambda < 0.9$ at $2\,\sigma$ with a peak at $\Lambda = 0.4$. However, the estimated value of Λ in Kochanek (1995) is sensitive to the detection threshold which is not well understood, and thus it can not be considered very seriously at the present stage.

It has been recognized that the mean splitting of the lensed images alone is useful for studies of the dynamical properties of lens galaxies, but not for the measurement of Λ (Turner, Ostriker, & Gott, hereafter TOG84; Fukugita et al. 1992). However, when the mean separation is used together with other information such as the lens redshift, the lens magnitude, and the velocity dispersion of the lens galaxy, then the mean separation does become sensitive to Λ (Paczynski & Gorski 1981; Gott et al. 1989; Kochanek 1992; Miralda-Escude 1991). In this *Letter*, we will try to measure Λ using a method which we call the “lens parameter method”, which is basically similar to those discussed in the above references.

2. Lens Parameter Method

The commonly observed parameters for gravitational lenses are the lens redshift (z_L), the source redshift (z_S), the mean deflection of the lensed object (or similarly the critical radius (θ_{crit})), the lens magnitude (m_L), and the source magnitude (m_S). For some systems, one or two of these observational parameters may be missing.

How sensitive is θ_{crit} to Ω_m and Λ for a given set of z_L, z_S, and m_L? To calculate θ_{crit}, we will adopt the singular isothermal sphere (SIS) model for the lens, along with the filled-beam approximation (see section 4), and the Faber-Jackson relation (Faber & Jackson 1976). The Faber-Jackson relation relates the velocity dispersion (σ) of E/S0 galaxies to their luminosities (L): $\sigma = \sigma_*(L/L_*)^\beta$. Here, we adopt $\sigma_* = 225km/sec$, $\beta = 0.25$ and $M_{*B_T} = -19.9 + 5log(h)$, taken from Kochanek (1992,1995).

Then $\theta_{crit}(z_L, z_S, m_L)$ can be expressed as;

$$\theta_{crit} = 4\pi(\sigma_*/c)^2\, d(z_S, z_L)(d(0, z_L)(1 + z_L)^2)^{4\beta}/d(0, z_S)\, 10^{-0.8\beta(m-M_*-K(z)-E(z)-25)} \qquad (1)$$

where $d(z_1, z_2)$ is the angular size distance between the redshifts z_1 and z_2 in Mpc, m is the total apparent magnitude of the lens galaxy, K(z) and E(z) are the K-correction and the evolutionary correction, respectively, for the lens galaxy. The (E+K) correction is important, and to calculate it we will use the 1 Gyr burst model of Bruzual & Charlot (1993) at the formation redshift $z_{for} = 10$. This (E+K) correction is consistent with the results from the HST Medium Deep Survey (MDS) on the evolution of the luminosity function of elliptical galaxies (Im et al. 1996), which shows a brightening in luminosity by about 1 magnitude looking back to $z \sim 1$.

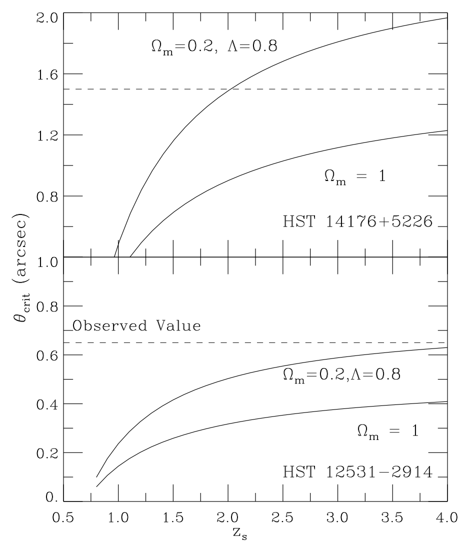

Figure 1: The θ_{crit}-z_S relation for two gravitational lenses HST14176+5226 and HST 12531-2914, assuming two different sets of cosmological parameters, i.e., ($\Omega_m = 1, \Lambda = 0$) and ($\Omega_m = 0.2, \Lambda = 0.8$). Observed values for θ_{crit} are indicated with the dashed lines.

Fig. 1 shows the $\theta_{crit} - z_S$ relation for the strong gravitational lens systems HST 12531-2914 and HST14176+5226, taking parameters from Ratnatunga et al. (1995). The two curves show the model predictions under the adoption of different cosmological parameters, and the horizontal line shows the observed value (the source redshift is unknown). The value of θ_{crit} is quite sensitive to Λ when values are known for z_L, z_S, and m_L. However, the uncertainty in the prediction is about a factor of $10^{0.15}$, which arises mainly from the uncertainties in the Faber-Jackson relation and in the apparent lens magnitude. Hence, a single lens system such as HST 12531-2914 cannot be used alone to measure Λ. In order to set a useful limit on Λ with this method, a sample of at least five lenses is required (e.g., see Kochanek 1992).

In order to combine the information on cosmological parameters from all available lenses, we therefore construct a likelihood function which is the product of the probability of each lens having the observed value of θ_{crit} for the given values of z_S, z_L, m_L and the cosmological parameters. This probability $p_i(\theta_o; z_L, z_S, m_L, \Omega_m, \Lambda)$ is defined as,

$$p_i(\theta_o; z_L, z_S, m_L, \Omega_m, \Lambda) \sim \int G(\theta_o, \theta_{crit}(z, z_S, m(z)), \sigma_\theta) G(z, z_L, \sigma_{z_L}) G(m(z), m_L, \sigma_{m_L}) dz \quad (2)$$

where σ_θ is the dispersion in the predicted $log_{10}(\theta_{crit})$ due to the uncertainty arising from the Faber-Jackson relation, together with other minor uncertainties, σ_{z_L} is the uncertainly in z_L, and σ_{m_L} is the uncertainty in m_L. $G(x, x_0, dx)$ is the Gaussian function with the mean of x_0 and the dispersion of dx. We adopt $\sigma_\theta = 0.14$ (or in terms of magnitude, $\sigma \simeq 0.7$) which is a combination of the uncertainties arising from the Faber Jackson relation ($0.65 \times 2 = 0.13$: de Zeeuw & Franx 1991), M_* ($0.3 \times 0.2 = 0.06$: Marzke et al. 1994; Loveday et al. 1992), and the E+K correction ($0.5 \times 0.2 = 0.1$: Im et al. 1996). When z_S is not available (HST12531-2914), we also integrate Eq.(2) over z_S, assuming a uniform distribution in redshift space.

Finally, the likelihood function can be written

$$L = \prod_j p_{j,norm}(\theta_j; z_{L,j}, z_{S,j}, m_{L,j}) \tag{3}$$

where $p_{j,norm}$ is the normalized probability of Eq.(2).

We did not adopt the $(3/2)^{0.5}$ factor (hereafter TOG factor) which was suggested by TOG84 in order to account for the possible difference between the velocity dispersion of the underlying dark matter and the luminous material. Recent studies show that this factor is not necessary (Kochanek 1993,1994; Breimer & Sanders 1993; Franx 1993). Independently, we also checked the necessity of the TOG factor by considering the mean image splittings (see Section 4).

The advantage of this method over the previous methods is the explicit use of the lens magnitude and the E+K correction, of which the latter has been observationally constrained only recently (Im et al. 1996; Pahre, Djorgovski, & de Carvalho 1996; Bender, Ziegler, & Bruzual 1996; Barrientos, Schade, & Lopez-Cruz 1996). These measurements enable us to make a reasonably good estimate of the dynamical properties of each lens galaxy. The probability of each individual lens having its unique configuration can then be calculated based on these individual properties, so that we do not have to use statistical measurements (e.g., the luminosity function) which may decrease the dependence on the cosmological parameters when they are averaged over large numbers of objects. Although our method is, in principle, not as sensitive to the value of Λ as is the lens number count method (e.g., Fukugita et al. 1992), the latter method is possibly subject to greater uncertainties (see section 4). Our method is slightly more susceptible to a small change in one of the input parameters, but, in common with the lens redshift method, we have a smaller number of parameters than the lens count method. In this respect, our method has an edge over the latter. In particular, the properties of lens galaxies at high redshift ($z \gtrsim 1.5$) are highly uncertain. They could be dusty enough that the result from the lens counts might be biased against the non-zero Λ model (Fukugita & Peebles 1995). In contrast, the lens parameter method uses lensing galaxies which lie at $z \lesssim 1$ (see section 3), and the method is thus less affected by the unknown properties of high redshift galaxies.

3. Sample Selection

Gravitational lenses are selected using the following criteria.

1) the strong lensing must be caused by a single galaxy lens. For example, we do not include 2016+112 in our sample since there are two lensing galaxies in this system. Also, we have excluded lens systems which are clearly influenced by strong perturbations due to cluster potentials (e.g., 0957+561, B1422+231).

2) it must be known that the lens galaxy is likely to be elliptical. For example, we do not include B0218+357 in our study since there is good evidence that the lensing galaxy is a spiral or a late-type galaxy (Patnaik et al. 1993).

3) the apparent magnitude and the redshift of the lens galaxy must be known or estimated to reasonable accuracy. Accurate values for m_L and z_L are important for estimates of the dynamical properties of the lens galaxy.

4) For lens candidates that do not have a measured value for z_S, we select only those which show distinctive features such as rings or crosses.

We find that there are seven strong gravitational lenses that meet these selection criteria in the published literature, including objects found in our HST surveys (Table 1). B1422+231 is excluded from this list because of the possible cluster perturbation as well as the ambiguity in the lens redshift ($z_L = 0.64$ from Hammer et al. 1995 vs. $z_L \simeq 0.4$ from Impey et al. 1996). For MG0414+0534, there have been speculations that the source redshift of the system is $z \simeq 1.00$ (Burke 1990; Kochanek 1992), but it was later established to be $z_S = 2.64$ (Lawrence et al. 1995), suggesting that the $z \simeq 1$ measurement pertains to the lens galaxy (Surdej & Soucail 1994). We have analyzed the archived HST observations of this system and find a preliminary result of $R_{F675W} - I_{F814W} = 1.5 \pm 0.2$ for the lens galaxy (Ratnatunga et al. 1996), suggesting that $z_L = 1.2 \pm 0.4$, consistent with the previous estimates of $z_L = 1$, and hence we will adopt $z_L = 1.2 \pm 0.4$ for this system. Finally, we have subtracted a few tenths of a magnitude from some of the quoted lens magnitudes in the literature, in order to correct for the total apparent magnitude. When the uncertainty in the lens magnitude is not quoted in the relevant reference, errors of about 0.3 magnitude are assigned to these lens galaxies.

4. Results and Discussion

In Fig. 2, we present the relative likelihood of our measurement against Ω_m for two cases which are of cosmological interest: i) $\Omega_m + \Lambda = 1$ and ii) $\Lambda = 0$. Both likelihood functions are normalized with the maximum likelihood of case (i), and direct comparison of cases (i) and (ii) is

possible using Fig.2. When a flat universe is assumed (case (i)), we find that $\Lambda = 0.64^{+0.15}_{-0.26}$, and we exclude the $\Omega_m = 1$ model with 97 % confidence. Also, a universe with $\Lambda \gtrsim 0.9$ is excluded at the 95 % confidence level. If $\Lambda = 0$ is assumed (case (ii)), then $\Omega_m \simeq 0$ is favored. The difference in the likelihood function between a flat universe with $\Lambda = 0.64$ and an open universe with $0 < \Omega < 0.3$ is about $0.5 - 1$. Hence, a flat universe with non-zero Λ is favored over an open universe at 68 % – 82 % confidence.

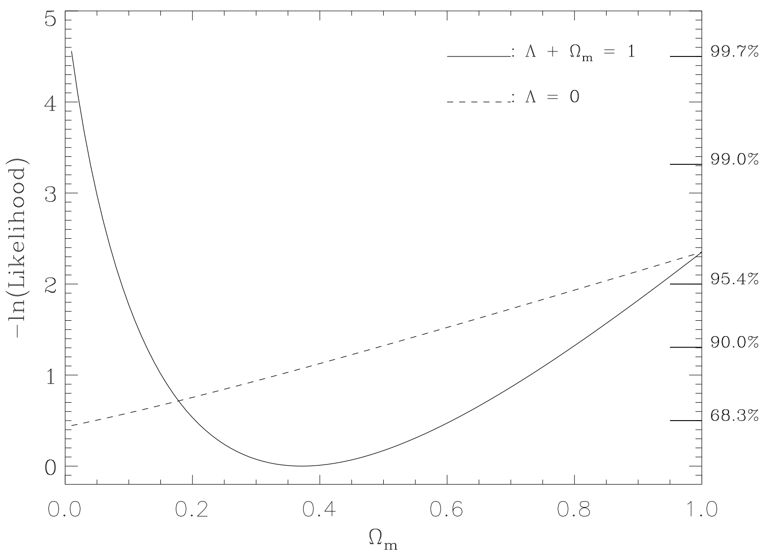

Figure 2: The likelihood of our measurement against Ω_m for two model universes: i) $\Omega_m + \Lambda = 1$ (solid line), and ii) $\Lambda = 0$ (dotted line). The relative likelihoods are normalized with the maximum of the likelihood of (i). Confidence limits are indicated on the right-hand side of the plot, assuming the χ^2 distribution for the likelihood.

Our result is only marginally consistent with the previous estimate of $\Lambda < 0.7$ based on the lens counts which strongly favored the zero Λ flat universe (Kochanek 1995; Maoz & Rix 1993). To see what might have caused the disagreement between our result and the previous results, we have investigated the possible systematic errors in our analysis and these are listed below:

1) the filled-beam approximation vs. the empty beam approximation

To relate redshift to distance, the filled beam approximation assumes that light rays propagate through smoothly averaged spacetime. In reality, spacetime is inhomogeneous, and therefore the filled beam approximation may not be correct (e.g., Fukugita et al. 1992). To see how our result could be affected by the filled beam approximation, the analysis was repeated adopting another extreme assumption, namely the empty beam approximation. We find that the latter approximation does not change our result significantly, but strengthens our finding slightly in favor of the non-zero Λ model.

2) Singular isothermal sphere vs. softened isothermal sphere.

We can also assume different mass models for the lens, rather than the SIS model. Recent

studies show that the SIS model may be too simple to adequately describe the mass of E/S0s (Lauer 1988; Krauss & White 1992), although the size of the core radius may be small enough to be negligible (Wallington & Narayan 1993). If the softened isothermal sphere is used, the predicted θ_{crit} will be a bit smaller than the predicted θ_{crit} with the SIS model. In Fig.1, this means that the predicted lines need to be shifted down along the θ_{crit} axis, making the $\Omega_m = 1$ flat model more inconsistent with the prediction. The adoption of the softened isothermal sphere model will thus strengthen our result.

3) Morphological misclassification

In our analysis, we have assumed that each lens galaxy is an E/S0. This assumption may be wrong, and to estimate the bias introduced by treating a spiral galaxy lens as an elliptical galaxy lens, we repeated our analysis with the inclusion of one known spiral lens system (B0218+357) treated as an E/S0 lens. This caused the result to be strongly biased in favor of the $\Lambda = 0$ model, because of the small predicted θ_{crit} of the spiral lens system, a result similar to the issue discussed in (2). Thus, if one of the seven lenses we used was actually a spiral galaxy rather than an elliptical, then the correction of it would only strengthen our result.

4) Wrong lens magnitude

Because the lens galaxy is much fainter than the lensed object in some cases, there is a possibility that the lens magnitudes are not well determined. Systematic overestimate of the lens magnitudes by more than 0.6 magnitude can bias the result against the $\Omega_m = 1$ model. Recent HST observations provide clues as to the accuracy of the ground-based estimates of lens magnitudes. The preliminary result by Falco (1995) from the HST observation of $0142-100$ gives an aperture magnitude of $R_{F675W} = 19.66 \pm 0.01$ for the lens galaxy, while the original measurement by Surdej et al. (1987) is R=19. For MG0414+0534, our preliminary analysis of the HST observation shows $I \simeq 21.4 \pm 0.15$ for the lens galaxy (Ratnatunga et al. 1996), agreeing with the previous estimate of $I \simeq 21.08 - 21.36$ from the ground (Schechter & Moore 1993). On the other hand, Impey et al. (1996) find $m_L = 21.5 \pm 0.3$ in V for the lens galaxy of B1422+231. The ground based estimate is $r = 21.8 \pm 0.6$ for this object (Yee & Ellingson 1994; Yee 1995). At $z \sim 0.4$, $V - r \sim 1$ for E/S0 galaxies, thus the observed ground-based lens magnitude for this system disagrees with the HST observation by about 1 magnitude. These three examples may indicate that the early ground-based measurements are not very accurate. Errors seem to go both ways, and hence there may be no systematic overestimate of the lens magnitudes. But there are only seven lenses in our sample, and it may be premature to say that there are no systematic errors in the lens magnitudes.

5) The TOG factor

With the TOG factor included, we find that the peak of the likelihood function shifts to the region where both Ω_m and Λ are very small, where our test becomes quite insensitive to the cosmological parameters. However, many studies have shown in different ways that the TOG factor is not needed (Kochanek 1995 and refs therein). In order to confirm earlier findings, we

analyzed the mean image deflections of the known lens systems with the known source redshifts. Using criteria (1) and (2) described in section 3, we find that there are 11 lens systems available for this analysis (see table 1 in Keeton & Kochanek 1995). For these systems, we calculated the ratio $\theta_{crit,obs}/ < \theta_{crit}(z_S) >$. Since $< \theta_{crit}(z_S) >$ is fairly independent of the cosmological parameters (TOG84; Fukugita et al. 1992), the average of 11 $\theta_{crit,obs}/ < \theta_{crit}(z_S) >$ values will be about 1 if the TOG factor is not necessary and about 1.5 if the TOG factor is appropriate. We find an average value of 1.0 ± 0.1, confirming that the TOG factor is not necessary. In order to test the TOG factor independently, the study of strong lenses at low redshift ($z < 0.1$) might be fruitful, since their lens parameters are then insensitive to the cosmological parameters. An optical survey which covers a large fraction of sky (e.g., SDSS) should be able to find a statistically significant number ($\sim$ 100) of such lenses.

6) Cluster Perturbation

Since elliptical galaxies preferentially live in a cluster environment, the gravitational potential of the lens may include a cluster component. The strong cluster perturbation generally increases the mean image deflection, and hence we tried to exclude such lenses from our study (See section 3-(1)). Nevertheless, we can not completely exclude the possibility that some of the lenses in our sample include a considerable amount of cluster perturbation. If that has happened, our result could be biased against the $\Omega_m = 1$ universe. To understand the possible contribution to the image splitting from the cluster potential, detailed modeling of the lens systems is desired using high resolution images from the HST, or else radio observations.

7) Source redshift for HST14176+5226

Crampton et al. (1996) have recently published a tentative source redshift for the lens system HST14176+5226. A strong emission line is found at 5324 $\AA$, along with a possible weak emission feature at 6822 $\AA$. The strong emission line is very likely to be Ly α at $z = 3.39$ if the weak emission feature at 6822 $\AA$ is real, and the latter can then be identified as CIV 1549. If the 6822 $\AA$ feature is not real, then the source object could be located at a redshift lower than $z_S = 3.4$. If $z_S < 3.4$ for the HST14176+5226, then the predicted θ_{crit} will be reduced. This would bring the peak of the likelihood function toward the large Λ value, strengthening the result in favor of the non-zero Λ model (see Fig. 1) .

8) E+K correction

The adopted E+K correction assumes a formation redshift of $z_{for} = 10$ with a 1 Gyr burst of star formation. We find that the E+K correction is most sensitive to the value of z_{for}, and insensitive to the other parameters. If we adopt $z_{for} > 10$, then the result changes insignificantly towards the zero Λ model. When $z_{for} < 10$, the result changes in favor of the non-zero Λ model, and the change is significant when $z_{for} < 2$. If $z_{for} = 1.5$, Λ could be as large as $\Lambda \simeq 0.8$.

If our result is an overestimate in the value of Λ, then there must have been a large systematic overestimate of the lens magnitudes and/or there are strong cluster perturbations. On the other

hand, if the lens counts have led to an underestimate in the value of Λ, then that could have been caused by: i) the dusty nature of high redshift elliptical galaxies (see section 2 for more discussion), ii) a decrease in the number density of ellipticals as a function of look-back time, as expected if most elliptical galaxies were created via major merging events (Im et al. 1996,1997; Baugh, Cole, & Frenk 1996; Kauffmann, Charlot, & White 1996), and/or iii) other uncertainties in the properties of lens galaxies, such as the LF and the dynamical properties of the low mass ellipticals. Future HST observations of faint galaxies, as well as the accumulating redshift data from ground based telescopes, will hopefully put stringent constraints on elliptical galaxy evolution at $z > 1$. These data will possibly give us indications as to why the results from the lens counts have strongly favored the zero Λ model while our result strongly rejects the flat universe with $\Lambda = 0$. It is noteworthy that neither method strongly rejects the low Ω universe.

5. Conclusions

We have described and applied the lens parameter method to measure cosmological parameters using strong gravitational lenses. Using seven strong lenses each with an identified lens galaxy, we find that a model universe with $\Lambda \sim 0.65$ and low Ω is favored and that the flat model with $\Lambda = 0$ is excluded at greater than 95 % confidence. A universe with low Ω and $\Lambda = 0$ can be marginally excluded with respect to the flat universe with a non-zero Λ at 68 % — 82 % confidence. Our result is not biased in favor of a non-zero Λ model due to any conceivable systematic errors, except for possible strong perturbations from cluster potentials, and systematic overestimate of the lens magnitudes. Future HST observations should uncover new lens systems with measurable lens properties suitable for this kind of study, and they should also provide a better understanding of the known lens systems. We should therefore be able to get a stronger constraint on Λ in the near future.

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TABLE 1

List of Gravitational Lenses

Name	m_L(total)	z_L	z_S	$\theta_{crit}('')$	References
HST14176+5226	I=19.71 $\pm$ 0.05	0.81	3.4(?)	1.51	Ratnatunga et al. (1995), Crampton et al. (1996)
HST12531-2914	I=21.82 $\pm$ 0.05	0.7 $\pm$ 0.1	< 5	0.65	Ratnatunga et al. (1995)
PG1115+080	R=18.36 $\pm$ 0.3	0.29	1.72	1.10	Weymann et al. (1980), Kristian et al. (1993)
MG1654+1346	R=18.4 $\pm$ 0.3	0.25	1.74	1.05	Langston et al. (1989)
CLASS1608+656	R=19.7 $\pm$ 0.3 (K$\simeq$ 16)	0.63	1.39	1.05	Myer et al. (1995a), Fassnacht et al. (1996)
0142-100	R=19.36 $\pm$ 0.10	0.49	2.72	1.10	Surdej et al. (1987), Falco (1995)
MG0414+0534	I=21.22 $\pm$ 0.15	1.2 $\pm$ 0.4	2.64	1.05	Hewett et al. (1992), Lawrence et al. (1995), Schechter & Moore (1992)
B0218+357*	R=20.0 $\pm$ 0.3	0.68	0.94	0.35	Patnaik et al. (1993)

*Spiral galaxy lens

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